# Illiquidity Risk and Capital Structure of Financial Institutions 

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#### Abstract

Following the framework for credit risk developed in Morris and Shin (2016), I construct a model for the financial structure decision of a bank in light of illiquidity and insolvency risk. Numeric analysis shows that the tax benefit of short-term debt can be outweighed by the negative effects of illiquidity risk for certain values of exogenous parameters, leading to a breakdown of the pecking order theory of financial structure. I qualitatively discuss an extension to a sequential signaling game framework similar to that of Noe (1988), as well as the policy implication that recent regulatory requirements concerning liquidity are sensible but imperfect.


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## Chapter 1

## Introduction

### 1.1 Background

Bank runs are an unfortunate but real feature of the financial system. Historically, in the era of the Great Depression, they occurred when many depositors at a retail bank simultaneously withdrew their cash deposits. Due to the nature of fractionalreserve banking ${ }^{1}$ most of the bank's assets were composed of loans that could not be immediately recalled for cash - that is, they were illiquid assets. Therefore, the mass withdrawal of deposits often resulted in the failure of the bank because it did not have enough cash to cover all the withdrawals. In modern times, with the advent of deposit insurance, such runs by retail investors are rare.

However, bank runs have simply taken on a different form. When the bank's short-term debt matures, creditors in the financial markets who hold the short-term debt must choose whether to roll over their holdings, with the goal of maximizing their expected payoffs. Although holding the bank's short-term debt is profitable when the bank's fundamentals are good, there is a risk that other creditors will run and not roll over their holdings. If this happens, the bank must pay off these

[^0]other creditors, and if there is not enough cash to cover these withdrawals, the bank becomes illiquid, leaving the debt devalued and unprofitable to hold. Thus, there is an element of strategic interaction: individual creditors must consider the likelihood that other creditors will choose to run. Negative signals about the bank's prospects may therefore change the creditors' beliefs about what the other creditors will do, possibly causing some creditors to run. If enough creditors decide to "run," the bank may fail due to an inability to raise enough cash to pay all the creditors.

Thus the danger of bank runs to banks, termed illiquidity risk, stems from the depletion of the liquid cash assets of a bank. It is related but not equivalent to insolvency risk, which refers to the danger that the value of a bank's assets falls below the value of its liabilities, rendering its net worth negative and often triggering bankruptcy. Illiquidity can occur without insolvency and vice versa; in addition, illiquidity can easily lead to insolvency if, for example, the bank must sell its nonliquid assets at a deep discount in a "fire sale" in order to raise enough cash to pay off withdrawing investors. Both illiquidity and insolvency risk played large roles in the financial crisis of 2007-2008: the withdrawal of short-term funding by many institutional investors resulted in a lack of short-term credit in debt markets, causing severe financial distress for many banks. ${ }^{2}$

In light of the illiquidity risk issues posed by short-term debt, why do banks use short-term debt financing at all? The answer is that it can be cheaper than other forms of financing, including long-term debt, which often carries higher interest rates; and equity, which does not confer a benefit from the tax-deductibility of debt interest. Therefore, there exists a trade-off between the tax advantages and illiquidity disadvantages of short-term debt.

[^1]
### 1.2 Literature Review

### 1.2.1 Financial Structure

All firms must finance their assets and investments with either debt or equity, and the specific combination of debt and equity used by a firm is called its financial structure. The problem of optimal corporate financial structure has been well studied. The fundamental work on this subject is Modigliani and Miller (1958), which states that under the assumption of perfect capital markets, the value of a firm is independent of its financial structure. In other words, in the absence of market frictions, the firm's choice between debt and equity does not matter for the firm. However, distortions such as taxes, bankruptcy costs, and asymmetric information have different effects on debt and equity; for example, as previously mentioned, the tax-deductibility of debt interest favors debt over equity. Thus there is a meaningful trade-off between debt and equity, implying that there is an optimal financial structure comprising both debt and equity that maximizes firm value.

Since this fundamental result, two main theories have developed to explain how firms choose between debt and equity financing: the trade-off theory and the pecking order theory. Frank and Goyal (2007) as well as Harris and Raviv (1999) provide a survey of the literature on these two theories. The trade-off theory posits that the firm's choice of financial structure results from optimizing the trade-off between the tax-deductibility of interest, which is a benefit of debt, and bankruptcy costs if the bank is unable to pay off its liabilities, which is a cost of debt. Kraus and Litzenberger (1973) formalize this notion in a state-preference framework. Essentially, the trade-off theory shows that the debt-equity indifference result of Modigliani and Miller fails when the assumption of perfect markets is removed.

The pecking order theory of debt, proposed by Myers (1984) and Myers and Majluf (1984), postulates that firms prefer a "pecking order" of internal financing,
debt financing, and then equity financing because of asymmetric information. Because the firm knows more about its own finances and prospects than outside investors do, investors assume that a firm that issues equity must perceive itself to be overvalued and is therefore seeking to capitalize on this overvaluation, so they are less willing to provide financing to the firm. Thus the firm prioritizes sources of financing that are less sensitive to information asymmetries, namely internal financing and debt.

Noe (1988) introduces strategic behavior into the pecking order theory via a sequential signaling game framework. He shows that when an information asymmetry exists - that is, insiders have perfect information regarding the firm's cash flows, but security buyers do not - the pecking order theory holds as expected. However, the introduction of residual uncertainty for insiders results in a breakdown of the pecking order.

More recently, Fulghieri, Garcia, and Hackbarth (2013) show that under asymmetric information between insiders and outsiders and certain other conditions, the pecking order breaks down and equity can be preferred to debt. Bayar, Chemmanur, and Liu (2015) examine financial structure in an environment with short sale constraints and heterogeneous behavior among outsiders, finding, among other conclusions, that sufficient optimism of outsiders can invert the traditional pecking order.

### 1.2.2 Bank Runs

The possibility of bank runs by creditors is also well established in the literature. The foundational paper by Diamond and Dybvig (1983) shows that even if the fundamentals of the bank are such that the bank would be solvent if all creditors were to continue investing, the bank is still vulnerable to a bank run: a coordination failure in which creditors withdraw their funding out of fear that other creditors will do the same. Both a bank run and continued re-investment are shown to be Nash equilibria. Bryant (1980) uses a different approach, employing Samuelson's pure consumption-
loans model. Postlewaite and Vives (1987) provide a framework for reducing the Diamond and Dybvig model to a unique Nash equilibrium in which there is a positive probability of a bank run. Similarly, Goldstein and Pauzner (2005) use fundamentals to select an equilibrium and compute the ex-ante probability of a bank run.

Morris and Shin (2010) and Morris and Shin (2016) develop a framework for decomposing bank credit risk into insolvency risk and illiquidity risk. Using the global game framework developed by Carlsson and van Damme (1993), which introduces random noise into the players' observations of the incomplete information game to determine a unique equilibrium, Morris and Shin show that illiquidity risk is characterized by threshold-type behavior by short-term debt holders: there exists a value for the interim asset return, termed the "run point", above which all creditors roll over and below which all creditors run. Liang, Lutkebohmert, and Xiao (2014) and Liang, Lutkebohmert, and Wei (2015) develop a dynamic multi-period bank run model including both illiquidity and insolvency risk using a structural credit risk modeling approach. They effectively extend Morris and Shin (2010) to multiple periods and obtain similar conclusions.

He and Xiong (2012) develop a dynamic continuous-time model of bank runs. The time-varying fundamental and the bank's staggered debt structure lead to a unique threshold equilibrium for the creditors' rollover decision, a mechanism different from the global game framework of noisy private information.

Eisenbach et al. (2014) combine elements of both financial structure and bank runs in order to analyze how the stability of a bank, in terms of its ability to survive "stress events," depends on various balance-sheet characteristics. Their modeling framework consists of the balance sheet of a bank, composed of safe assets, risky assets, short-term debt, long-term debt, and equity. They assume that the risky asset is partially illiquid and that short-term debt holders may choose to run, leading to illiquidity risk. However, while the bank's balance sheet is endogenous and the
focus of their comparative statics analysis, their model treats creditor behavior and their rollover decision as exogenous. Their results include the finding that increased debt makes the bank more susceptible to bank runs and illiquidity risk, and the finding that lengthening the maturity structure of the bank's debt-that is, substituting some short-term debt for long-term debt-reduces the bank's vulnerability to funding shocks, but also increases vulnerability to shocks to the values of its assets since long-term debt is more costly than short-term debt. They also examine policy implications, illustrating that "liquidity requirements can have competing effects on stability, making a bank more resilient to funding shocks but less resilient to shocks to the value of its risky assets."

### 1.3 Overview of the Paper

As discussed above, the existing literature thoroughly examines the issues of financial structure and bank runs. However, they generally do so only in isolation: papers on financial structure do not address the issues of bank runs and illiquidity risk, and papers on bank runs take the bank's ex-ante choice of financial structure as exogenous. Therefore, with this paper I aim to develop a framework for considering both the bank's choice of financial structure and the creditors' rollover decision together in order to provide a more comprehensive model of bank finances and bank-creditor interactions that considers their effects on each other 3 The results of my analysis have significant and timely implications for public policy, especially concerning the effectiveness and calibration of liquidity requirements developed in the wake of the financial crisis of 2007-2008.

[^2]To model illiquidity risk and the strategic behavior of short-term debt holders, I adapt the framework of Morris and Shin (2016), which takes the financial structure of the bank to be exogenous. My contribution is to endogenize the balance sheet and thus incorporate the bank's choice of financing structure ex-ante. This extension of the model can then examine how the bank's financial structure can influence creditor behavior.

My model largely follows Morris and Shin (2016), though there are some differences ${ }^{4}$ Morris and Shin derive explicit expressions for the probabilities of illiquidity and insolvency and then use these in their derivation of the result that creditors run if and only if the asset return falls below a "run point" threshold. I exploit their run point result to compute the expected payoffs to the bank and to its security buyers, though I relax their assumption of a uniform distribution for the asset return. I mostly preserve the components of the balance sheet of Morris and Shin, which are similar to those in Eisenbach et al. (2014); however, in order to examine the financing decision of the bank for a given investment, I restrict the amount of the investment-the risky asset - to be exactly equal to the amount of short-term debt. Finally, I assign a reasonable payoff structure to the model, preserving the assumption that the bank's failure from illiquidity will set both bank and investor payoffs to zero, while letting debt buyers gain control of the bank if the bank is solvent but cannot make the full debt repayment.

My hypothesis is that without taxes and credit risk, the bank is indifferent between debt and equity, but taxes and credit risk influence the preference for debt in opposite directions. Because of taxes, the bank prefers debt to equity when there is no illiquidity risk, so the gain in payoff from using debt financing instead of equity financing, or the "debt savings," is positive. However, when illiquidity risk is introduced, it increases the cost of debt, so debt savings decrease. My goal is to characterize

[^3]the conditions under which the bank prefers debt financing to equity financing (that is, debt savings are positive) and analyze the comparative statics effects of exogenous parameters on the interest rate of the bank's debt and the debt savings.

I find that the trade-off between debt and equity exists as hypothesized, and that the bank's preference between debt and equity depends on the values of exogenous parameters. Specifically, debt savings becomes negative if the bank has too little liquidity, has too little net assets, has too large of an investment, has too low of an expected asset return, or faces a very attractive "outside option" that short-term debt buyers can run to, because debt buyers are more enticed to run due to the weak finances of the bank relative to the outside option. Interestingly, I also find that at high levels of net assets or expected asset return, the loss of "counterfactual equity" due to illiquidity risk, or the payoff a solvent bank would have received had it not been illiquid, becomes significant enough to make debt financing less attractive than equity financing. The existence of parameter sets at which equity is preferred to debt implies that the pecking order theory of financial structure breaks down, though it is consistent with the trade-off theory. These results have important policy implications, as regulatory efforts to keep credit markets functioning during a crisis, reduce the risk of bank runs, and improve the stability of the financial system are particularly concerned with preventing extremely adverse values of the exogenous parameters I examine. In particular, I show that the liquidity coverage ratio (LCR) is a sensible but not comprehensive way to reduce illiquidity risk.

The rest of this paper is organized as follows. In Chapter 2, I construct a model of the bank's balance sheet and the payoffs to the bank and its creditors, adhering closely to the model of Morris and Shin (2016). In Chapter 3, I analytically derive an implicit solution of the model and confirm the conclusion of Modigliani and Miller: in the absence of taxes and illiquidity, the bank is indifferent between debt and equity. However, in their presence, the trade-off between taxes and illiquidity risk leads to
a strict preference between debt and equity, in accordance with the trade-off theory of financial structure. Unfortunately, even if I assume a uniform distribution for the asset return as in the literature, e.g. Morris and Shin (2016), deriving a closedform solution is intractable. Therefore, I use numeric analysis to further examine this trade-off and perform comparative statics analysis in Chapter 4. In Chapter 5. I qualitatively discuss an extension of my model to incorporate the information asymmetry of the sequential signaling game framework of Noe (1988) and thus the pecking order theory of financial structure. Finally, I discuss the policy implications of my model in Chapter 6. Chapter 7 contains concluding remarks.

## Chapter 2

## Model

Following the model of Morris and Shin (2016), I consider the behavior of a bank and buyers of the bank's securities in a three-period model. To simplify the analysis, I restrict my attention to the choice between short-term debt and equity, excluding long-term debt as a financing option. Nevertheless, long-term debt is interesting: while it confers tax benefits and is not subject to illiquidity risk since its long maturity implies that buyers cannot run in the interim period, it is more expensive than shortterm debt; see Section 6.1.2 for a qualitative discussion of an extension of the model to include long-term debt.

In the ex-ante period, the bank undertakes an investment opportunity $Y$ and chooses to finance it by issuing either short-term debt or equity, which is bought by buyers in the financial markets. In the interim period, if the bank chose to finance with short-term debt, the buyers can choose to run; i.e., not roll over their holdings. This option to run introduces illiquidity risk for short-term debt. In the ex-post period, the investment yields some return and both the bank and its creditors collect payoffs. If the bank finances with short-term debt, the interest paid by the bank is tax-deductible at tax rate $T$. There is no discounting of the payoffs. The structure
of the balance sheet and the payoffs are similar to those in Morris and Shin (2016), and they define a logical simple model for corporate financial structure.

### 2.1 Balance Sheet

Table 2.1: Balance Sheet

| Assets | Liabilities \& Equity |
| :--- | :--- |
| Cash $M$ | Long-term debt $L$ |
| Non-liquid asset $A$ | Equity $E$ |
| Risky investment $Y$ | New financing: short-term debt or new equity |

The balance sheet of the bank is summarized in Table 2.1. On the asset side, the bank has liquid and non-risky cash $M$, a non-liquid and non-risky asset $A$, and the non-liquid risky investment $Y$. The presence of the non-liquid asset $A$ allows for the existence of illiquidity risk on short-term debt: because $A$ and $Y$ are not liquid, they cannot be used to pay withdrawals of short-term debt in the event of a run. In the terminology of Morris and Shin (2010, 8), this means that no cash at all can be raised from the risky asset to pay withdrawals: $\psi=0$. On the liabilities and equity side, the bank has long-term debt $L$, initial equity $E$, and some new financing for $Y$ in the form of either short-term debt or new equity. To focus on the bank's decision between short-term debt and equity, I assume that long-term debt $L$ does not mature or change in value during the ex-ante, interim, or ex-post periods. In accordance with my focus on short-term debt, from this point forward, unless otherwise stated, any reference to "debt" will refer to short-term debt and any reference to "debt buyers" will refer to short-term debt buyers.

Define the liquidity coverage ratio $\lambda$ as the portion of short-term debt $Y$ that can be repaid with (is "covered" by) liquid cash assets $M$ To allow for illiquidity risk,

[^4]I assume that this ratio is less than 1, so if all debt buyers run in the interim period, not all of them can be paid, since the only liquid asset available for payment is cash $M$. Otherwise, the bank would always survive a run and there would be no illiquidity risk. In summary,

$$
\lambda=\frac{M}{Y}<1
$$

This quantity will play a role in determining the behavior of short-term debt buyers in the presence of illiquidity risk. It is also the subject of active policy discussion following the financial crisis of 2007-2008.

Let the gross return on the risky asset in the ex-post period be $\theta_{2}$, whose distribution depends on the value of $\theta_{1}$, its expectation in the interim period. Also, let the distribution of $\theta_{1}$ depend on the value of $\theta_{0}$, its expectation in the ex-ante period.

### 2.2 Payoffs in the Absence of Illiquidity Risk

For this section, I assume that there is no illiquidity risk. First, I make an assumption concerning the terms at which the bank can finance its investment: the buyers of the bank's securities expect to earn zero expected profits, due to Bertrand-like competitive bidding in the financial markets. Noe (1988) gives a justification for this argument in a game-theoretic context. This break-even assumption determines the terms at which the bank can finance its investment, allowing for a comparison of the payoffs to the bank under debt and equity financing.

If the bank chooses to finance with short-term debt, then it is liable for an interest payment in the ex-post period. Specifically, if the bank is solvent in the ex-post period, then it must repay debt buyers the principal with interest $-Y(1+r)$. If this repayment is made, the tax-deductibility of interest means that the bank saves an amount $\operatorname{Tr} Y$ on its taxes, so the net amount that the bank pays out is effectively $Y(1+(1-T) r)$. However, if the bank is solvent ex-post but cannot make the full payment, then the
bank defaults and short-term debt buyers gain control of the remaining balance sheet of the bank. If this happens, the debt buyers receive a payoff equal to the remaining equity value of the bank, the bank receives a payoff of 0 , and there is no tax benefit. If the bank is insolvent, both the bank and the buyers receive a payoff of 0 . By the break-even assumption, the rate of interest $r$ is chosen such that the buyers expect to break even.

If the bank chooses to finance with equity, it sells equity shares of itself such that the buyers own a portion $\alpha$ of the bank. If the bank is solvent in the ex-post period, both the bank and its equity buyers receive payoffs equal to their shares of the equity value of the bank; otherwise, both parties receive payoffs of 0 . The share $\alpha$ is chosen such that the buyers expect to break even.

Under either type of financing, the bank is solvent in the ex-post period if:

$$
\begin{aligned}
M+A+\theta_{2} Y-L & \geq 0 \\
\theta_{2} & \geq-\frac{M+A-L}{Y}
\end{aligned}
$$

Under debt financing, the bank is solvent but cannot make the full payment to debt buyers in the ex-post period if:

$$
\begin{aligned}
0 & \leq M+A+\theta_{2} Y-L<Y(1+r) \\
-\frac{M+A-L}{Y} & \leq \theta_{2}<1+r-\frac{M+A-L}{Y}
\end{aligned}
$$

Under debt financing, the bank is solvent and can make the full payment to debt buyers in the ex-post period if:

$$
\begin{aligned}
M+A+\theta_{2} Y-L & \geq Y(1+r) \\
\theta_{2} & \geq 1+r-\frac{M+A-L}{Y}
\end{aligned}
$$

### 2.2.1 Debt Financing Payoffs

Based on the preceding discussion, if the bank finances with debt, then the payoff to the debt buyers as a function of $\theta_{2}$ is as follows:

$$
\text { Payoff }_{\text {Debt, Buyers }}\left(\theta_{2}\right)= \begin{cases}0 & \text { if } \theta_{2}<-\frac{M+A-L}{Y}  \tag{2.1}\\ M+A+\theta_{2} Y-L & \text { if }-\frac{M+A-L}{Y} \leq \theta_{2}<1+r-\frac{M+A-L}{Y} \\ Y(1+r) & \text { if } \theta_{2} \geq 1+r-\frac{M+A-L}{Y}\end{cases}
$$

The payoff to the bank as a function of $\theta_{2}$ is as follows:
Payoff $_{\text {Debt, Bank }}\left(\theta_{2}\right)= \begin{cases}0 & \text { if } \theta_{2}<1+r-\frac{M+A-L}{Y} \\ M+A+\theta_{2} Y-L-Y(1+(1-T) r) & \text { if } \theta_{2} \geq 1+r-\frac{M+A-L}{Y}\end{cases}$

### 2.2.2 Equity Financing Payoffs

Based on the preceding discussion, if the bank finances with equity, then the payoff to the equity buyers as a function of $\theta_{2}$ is as follows:

$$
\text { Payoff }_{\text {Equity, Buyers }}\left(\theta_{2}\right)= \begin{cases}0 & \text { if } \theta_{2}<-\frac{M+A-L}{Y}  \tag{2.3}\\ \alpha\left(M+A+\theta_{2} Y-L\right) & \text { if } \theta_{2} \geq-\frac{M+A-L}{Y}\end{cases}
$$

The payoff to the bank as a function of $\theta_{2}$ is as follows:

$$
\text { Payoff }_{\text {Equity, Bank }}\left(\theta_{2}\right)= \begin{cases}0 & \text { if } \theta_{2}<-\frac{M+A-L}{Y}  \tag{2.4}\\ (1-\alpha)\left(M+A+\theta_{2} Y-L\right) & \text { if } \theta_{2} \geq-\frac{M+A-L}{Y}\end{cases}
$$

### 2.2.3 Graphical Comparison of Debt and Equity Payoffs

Figures 2.1 and 2.2 graph the payoffs to buyers and the bank under debt and equity financing in the absence of illiquidity risk. Payoffs under debt financing are in solid blue and the payoffs under equity financing are in dashed red.

Figure 2.1: Payoffs to Buyers
Buyer Payoff as a Function of Asset Return $\boldsymbol{\theta}_{2}$
(No Illiquidity Risk)


At this stage, I assume that the bank can procure financing of whichever type it likes at the break-even terms, so the security buyers do not choose between the two payoffs. Thus, the graphical comparison of payoffs to debt and equity buyers is only for clarity and completeness. The comparison for the bank, meanwhile, is more interesting. ${ }^{2}$ Observe that both payoffs are linear in $\theta_{2}$ above certain critical values

[^5]Figure 2.2: Payoffs to Bank

of $\theta_{2}$, but the debt payoff has a higher slope and higher critical value. This has an appealing intuitive explanation: under debt financing, the bank keeps all profits once the debt buyers have been paid back, but is exposed to the added risk of default when $\theta_{2} \in\left(-\frac{M+A-L}{Y}, 1+r-\frac{M+A-L}{Y}\right)$. However, under equity financing, the bank must share all profits with equity buyers - hence the lower slope - but has no default risk. At low values of $\theta_{2}$, the bank prefers the equity payoff, but at higher values of $\theta_{2}$, debt financing yields a higher payoff. This holds even without considering the tax benefit of debt, which simply serves to increase the bank's payoff by a constant.

The goal of my analysis is to determine when the bank prefers debt over equity: i.e. when the expected payoff to the bank from debt financing is greater than the expected payoff from equity financing. From the graph, it is evident that at some values of $\theta_{2}$, debt is preferred, while at other values of $\theta_{2}$, equity is preferred; thus
it is not immediately clear how the expectations over $\theta_{2}$ of debt and equity payoffs compare. I find that with no tax savings and no illiquidity risk, the expectations are actually equal, regardless of the distribution of $\theta_{2}$, because the break-even condition adjusts the values of $\alpha$ and $r$ accordingly.

### 2.3 Payoffs in the Presence of Illiquidity Risk

The previous payoff analysis incorporated insolvency risk by setting payoffs to zero when the bank was insolvent in the ex-post period. Now I add illiquidity risk and examine how payoffs change.

### 2.3.1 Run Point

Suppose the bank finances its investment $Y$ with short-term debt. Morris and Shin $(2010,2016)$ argue that the behavior of short-term debt buyers is characterized by a threshold: all debt buyers run if $\theta_{1}<\theta_{0}^{*}$ and no debt buyers run if $\theta_{1}>\theta_{0}^{*}$ for some constant $\theta_{0}^{*}$ denoted the "run point." They justify this threshold-type behavior with a global game framework. Assume that debt buyers do not observe $\theta_{1}$ directly, but rather observe a noisy signal for $\theta_{1}$. In this game of incomplete information, there is a unique equilibrium in which debt buyers roll over if and only if their signal exceeds a critical value. Then, letting the magnitude of the noise tend to zero, this unique equilibrium predicts exactly the threshold-type behavior characterized by the existence of a run point $\theta_{0}^{*}{ }^{3}$

At this point, it should be noted that there are several differences between my model and that of Morris and Shin (2010, 2016). These differences are necessary and indeed desirable for examining the financial structure of the bank. Fortunately, the

[^6]run point result of Morris and Shin still holds, allowing me to characterize the behavior of short-term debt buyers according to a closed-form threshold for the interim asset return. This enables a clean consideration of illiquidity risk in the context of the bank's financing decision.

First, Morris and Shin use a uniform distribution for $\theta_{2}$ to derive their run point, whereas my model seeks to be more general. $\left.\right|_{4} ^{4}$ Morris and Shin (2003) extend the analysis to general distributions, so under some "mild regularity conditions on the smoothness of densities" (Morris and Shin 2010, 41) the same analysis should hold.

Next, I restrict the amount of short-term debt, denoted $S$ in Morris and Shin, to be exactly equal to the amount of the (debt-funded) risky investment $Y$, and add a non-risky but also non-liquid asset $A$. The purpose of the restriction on $S$ is to be able to examine the marginal financing decision of the bank as a choice between short-term debt and equity. Essentially, this restriction on $S$ in my model separates the risky asset $Y$ from Morris and Shin into a portion funded by short-term debt and a portion funded by other sources. The presence of the non-liquid asset $A$ in my model represents the portion funded by other sources, namely long-term debt and equity. $A$ preserves the possibility of a run by ensuring that the cash $M$ is not enough to cover the entire amount of the short-term debt if debt buyers run while still allowing the balance sheet to balance. The caveat is that $A$ is no longer risky as in Morris and Shin, but Morris and Shin (2010, 23-24) argues that their analysis extends to a general balance sheet with multiple risky assets. My model can therefore be interpreted as the special case of their generalized analysis where the return of the "risky" asset $A$ is constant $5^{5}$

[^7]Finally, I make a change to the payoff structure: if the bank is solvent but cannot repay its debt obligation, i.e. $\theta_{2} \in\left(-\frac{M+A-L}{Y}, 1+r-\frac{M+A-L}{Y}\right)$, the payoff to debt buyers is 0 in Morris and Shin but equal to the remaining equity of the bank $M+A+$ $\theta_{2} Y-L$ in my model. This change in payoffs enables my model to have a sensible baseline consistent with Modigliani and Miller (1958), as will be seen later: without illiquidity risk and taxes, debt and equity yield identical payoffs. Fortunately, Morris and Shin $(2010,8)$ state that allowing for "positive recovery rates" in the event of default-for example, the debt buyers gaining control of the bank-would not qualitatively change their analysis.

Now that the usage of the run point in my model has been justified, I now examine how the run point is derived. Let $r^{*}$ be a new independent parameter representing the return on an outside option that is available to debt buyers should they decide to run. Morris and Shin (2016, 9-10 and 2010, 11-13) set the expected return to the debt buyers from rolling over equal to the outside option:

$$
\theta_{0}^{*}=\theta_{1} \text { st. } \lambda r \mathbb{P}\left(\text { solvent } \mid \theta_{1}\right)=r^{*}
$$

where $\lambda$ is the liquidity coverage ratio. The intuition is that debt buyers receive return $r$ if and only if the bank is solvent and survives a run, the latter of which occurs with probability $\lambda$. Since the rollover decision occurs in the interim period, I condition on $\theta_{1}$. Observe that for this equation to hold, since probabilities are at most 1 it is necessary to have

$$
r \geq \frac{r^{*}}{\lambda}
$$

Intuitively, if $r$ is too low, the outside option will always be better than rolling over regardless of the solvency or liquidity situation of the bank. Because I assume that $\lambda<1$ to allow for liquidity risk, at the very least I must have $r>r^{*}$. That is, the interest rate offered by the bank must be, at the very least, greater than the return
on the outside option, or else the outside option would always yield a higher return than the bank's short-term debt and no one would buy the bank's debt.

In Morris and Shin (2010, 13), under their uniform distribution the run point is (in my notation):

$$
\theta_{0}^{*}=-\frac{M+A-L}{Y}+\sigma_{2}\left(\frac{r^{*} / r}{M / Y}-\frac{1}{2}\right)
$$

In my model, the corresponding run point is

$$
\begin{equation*}
\theta_{0}^{*}=\theta_{1} \text { st. } \frac{M}{Y} r \mathbb{P}\left(\left.\theta_{2} \geq 1+r-\frac{M+A-L}{Y} \right\rvert\, \theta_{1}\right)=r^{*} \tag{2.5}
\end{equation*}
$$

which corresponding condition

$$
\begin{equation*}
r \geq \frac{r^{*}}{M / Y} \tag{2.6}
\end{equation*}
$$

Assume $\theta_{2}=\theta_{1}+\sigma_{2} \varepsilon_{2}$, where $\varepsilon_{2} \sim N(0,1){ }^{6}$ Let $\Phi$ be the cumulative distribution function of $\varepsilon_{2}$ (the standard normal CDF). I then solve (2.5) explicitly:

$$
\begin{aligned}
\frac{M}{Y} r \mathbb{P}\left(\left.\theta_{2} \geq 1+r-\frac{M+A-L}{Y} \right\rvert\, \theta_{1}\right) & =r^{*} \\
\mathbb{P}\left(\left.\varepsilon_{2} \geq \frac{1+r-\frac{M+A-L}{Y}-\theta_{1}}{\sigma_{2}} \right\rvert\, \theta_{1}\right) & =\frac{r^{*} / r}{M / Y} \\
1-\Phi\left(\frac{1+r-\frac{M+A-L}{Y}-\theta_{1}}{\sigma_{2}}\right) & =\frac{r^{*} / r}{M / Y} \\
1+r-\frac{M+A-L}{Y}-\sigma_{2} \Phi^{-1}\left(1-\frac{r^{*} / r}{M / Y}\right) & =\theta_{1}
\end{aligned}
$$

Therefore I conclude that under a normal distribution for $\theta_{2}$, the run point is: 7

$$
\begin{equation*}
\theta_{0}^{*}=\left(1+r-\frac{M+A-L}{Y}\right)-\sigma_{2} \Phi^{-1}\left(1-\frac{r^{*} / r}{M / Y}\right) \tag{2.7}
\end{equation*}
$$

[^8]Observe that $\Phi^{-1}(x)$ is only defined for probabilities $x \in[0,1]$. Since all parameters are nonnegative, this implies that a necessary condition for $\theta_{0}^{*}$ to be well-defined is:

$$
\begin{equation*}
r \geq \frac{r^{*}}{M / Y} \tag{2.8}
\end{equation*}
$$

which is identical to condition (2.6).

### 2.3.2 Change in Payoffs Due to Illiquidity Risk

If the bank finances its investment with short-term debt, it is subject to illiquidity risk. By the preceding analysis, all debt buyers run if $\theta_{1}<\theta_{0}^{*}$ and no debt buyers run if $\theta_{1}>\theta_{0}^{*}$, where $\theta_{0}^{*}$ is defined by (2.5). Therefore, I assume that if $\theta_{1}<\theta_{0}^{*}$, the bank fails due to illiquidity in the interim period and both the bank and debt buyers receive a payoff of zero 8 With only insolvency risk, payoffs are set to 0 if the bank is insolvent; now, with illiquidity risk, payoffs are also set to 0 if the bank is illiquid. Payoffs are otherwise unchanged; in particular, if the bank is liquid in period 1 , the graphs in Section 2.2.3 still hold.

[^9]
## Chapter 3

## Analytic Solution of the Model

I now solve the model described in Chapter 2, A solution to the model defines an expression for the debt savings, which is the difference in expected payoffs for the bank between debt and equity financing. To derive a solution, I use the break-even condition to determine the interest rate $r$ and the equity share $\alpha$ and then compute an expectation of the bank's payoffs. I do this both in the absence of illiquidity risk and in the presence of illiquidity risk in order to highlight the effect of illiquidity risk.

In the derivations that follow, let $f\left(\theta_{1}\right)$ be the probability density function of $\theta_{1}$ and $g\left(\theta_{2} \mid \theta_{1}\right)$ be the conditional probability density function of $\theta_{2}$ given $\theta_{1}$.

### 3.1 Financing in the Absence of Illiquidity Risk

First, assume that there is no illiquidity risk. With this assumption, in my threeperiod model the interim period is not relevant because short-term debt buyers do not run. Observe that in the calculations that follow, the interim asset return $\theta_{1}$ is irrelevant and simply integrated over the entire real line. Only the final value $\theta_{2}$ for the asset return is relevant for payoffs and therefore for the break-even value of $r$ and the debt savings as well.

### 3.1.1 Debt Financing

First, assume that the bank chooses to finance using short-term debt. From (2.1), the ex-ante expected payoff received by the buyers is:

$$
\begin{aligned}
& \mathbb{E}[\text { Payoff }\text { Debt, Buyers }]= \\
& \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} \mathbb{E}\left[\text { Payoff } \mid \theta_{2}\right] \mathbb{P}\left(\theta_{2} \in d \theta_{2} \mid \theta_{1}\right)\right] \mathbb{P}\left(\theta_{1} \in d \theta_{1}\right) \\
&= \int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
&+Y(1+r) \int_{-\infty}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty} g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}
\end{aligned}
$$

The break-even condition requires that this be equal to $Y$, the initial investment:

$$
\begin{align*}
Y= & \int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& +Y(1+r) \int_{-\infty}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty} g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \tag{3.1}
\end{align*}
$$

which can also be written as

$$
\begin{align*}
& \int_{-\infty}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty} g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& \quad=\frac{1}{1+r}-\frac{1}{Y(1+r)} \int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \tag{3.2}
\end{align*}
$$

From (2.2), the ex-ante expected payoff to the bank is:

$$
\begin{align*}
\mathbb{E}\left[\text { Payoff }_{\text {Debt, Bank }}\right]= & \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} \mathbb{E}\left[\text { Payoff } \mid \theta_{2}\right] \mathbb{P}\left(\theta_{2} \in d \theta_{2} \mid \theta_{1}\right)\right] \mathbb{P}\left(\theta_{1} \in d \theta_{1}\right) \\
= & \int_{-\infty}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& -Y(1+(1-T) r) \int_{-\infty}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty} g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
= & \int_{-\infty}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& -\frac{Y(1+(1-T) r)}{1+r} \\
& +\frac{1+(1-T) r}{1+r} \int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \tag{3.3}
\end{align*}
$$

where the last equality follows from (3.2).

### 3.1.2 Equity Financing

Now assume that the bank chooses to finance using equity. From (2.3), the ex-ante expected payoff received by the buyers is:

$$
\begin{aligned}
\mathbb{E}[\text { Payoff Equity, Buyers }] & =\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} \mathbb{E}\left[\text { Payoff } \mid \theta_{2}\right] \mathbb{P}\left(\theta_{2} \in d \theta_{2} \mid \theta_{1}\right)\right] \mathbb{P}\left(\theta_{1} \in d \theta_{1}\right) \\
& =\alpha \int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}
\end{aligned}
$$

The break-even condition requires that this be equal to $Y$, the initial investment.
Dividing through by the double integral:

$$
\begin{equation*}
\alpha=\frac{Y}{\int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}} \tag{3.4}
\end{equation*}
$$

Equation (3.4) states that the portion of the bank owned by equity buyers is equal to the fraction of the expected value of the bank represented by the initial investment. From (2.4), the ex-ante expected payoff to the bank is:

$$
\begin{align*}
\mathbb{E}\left[\text { Payoff }_{\text {Equity, Bank }}\right] & =\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} \mathbb{E}\left[\text { Payoff } \mid \theta_{2}\right] \mathbb{P}\left(\theta_{2} \in d \theta_{2} \mid \theta_{1}\right)\right] \mathbb{P}\left(\theta_{1} \in d \theta_{1}\right) \\
& =(1-\alpha) \int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& =\int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}-Y \tag{3.5}
\end{align*}
$$

where the last equality follows from (3.4). Equation (3.5) states that the expected payoff to the bank under equity financing is the expected value of the bank's equity minus the expected value $Y$ of the buyers' share.

### 3.1.3 Debt Savings

Using equations (3.3) and (3.5), I obtain that the expected savings from using debt instead of equity financing are:

$$
\begin{align*}
(3.3)-(3.5)= & -\int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& +Y-\frac{Y(1+(1-T) r)}{1+r} \\
& +\frac{1+(1-T) r}{1+r} \int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
= & \left(1-\frac{1+(1-T) r}{1+r}\right) \\
& \times\left(Y-\int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}\right) \\
= & \left(\frac{T r}{1+r}\right) Y(1+r) \int_{-\infty}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty} g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
= & \operatorname{Tr} Y \mathbb{P}\left(\theta_{2} \geq 1+r-\frac{M+A-L}{Y}\right) \tag{3.6}
\end{align*}
$$

where the penultimate equality follows from (3.1). Since the bank pays interest $r Y$ to debt buyers if and only if $\theta_{2} \geq 1+r-\frac{M+A-L}{Y}$, equation (3.6) states that the expected savings from using debt instead of equity financing is equal to the tax rate multiplied by the expected interest paid by the bank. Observe that in the absence of taxes $(T=0)$, debt savings are exactly zero. This is the conclusion of the ModiglianiMiller theorem: in the absence of market imperfections such as taxes, the bank is indifferent between debt and equity.

### 3.2 Financing in the Presence of Illiquidity Risk

Now I reintroduce illiquidity risk, so short-term debt buyers may run in the interim period. From the discussion in Section 2.3.2, the only difference that illiquidity risk makes is that payoffs are zero when $\theta_{1}<\theta_{0}^{*}$, where the run point $\theta_{0}^{*}$ is defined by (2.5). Therefore, the only change to the break-even conditions and expected payoffs of the previous section is that for debt financing, the domain of integration over $\theta_{1}$ is now $\left(\theta_{0}^{*}, \infty\right)$. However, because the domain of integration over $\theta_{1}$ is now different for debt and equity, the expression for debt savings changes. For completeness, I include all expressions below.

### 3.2.1 Debt Financing

The corresponding equation for the break-even condition (3.1) is:

$$
\begin{align*}
Y= & \int_{\theta_{0}^{*}}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& +Y(1+r) \int_{\theta_{0}^{*}}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty} g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \tag{3.7}
\end{align*}
$$

The corresponding expression for the ex-ante expected payoff to the bank (3.3) is:

$$
\begin{align*}
& \int_{\theta_{0}^{*}}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
& \quad-\frac{Y(1+(1-T) r)}{1+r} \\
& \quad+\frac{1+(1-T) r}{1+r} \int_{\theta_{0}^{*}}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \tag{3.8}
\end{align*}
$$

### 3.2.2 Equity Financing

The equation for the break-even condition (3.4) is unchanged:

$$
\begin{equation*}
\alpha=\frac{Y}{\int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}} \tag{3.9}
\end{equation*}
$$

The expression for the ex-ante expected payoff to the bank (3.3) is unchanged:

$$
\begin{equation*}
\int_{-\infty}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}-Y \tag{3.10}
\end{equation*}
$$

### 3.2.3 Debt Savings

Using equations (3.8) and (3.10), I obtain that the expected savings from using debt instead of equity financing are:

$$
\begin{align*}
&(3.8)-(3.10)=-\int_{\theta_{0}^{*}}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
&+Y-\frac{Y(1+(1-T) r)}{1+r} \\
&+\frac{1+(1-T) r}{1+r} \int_{\theta_{0}^{*}}^{\infty}\left[\int_{-\frac{M+A-L}{Y}}^{1+r-\frac{M+A-L}{Y}}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
&-\int_{-\infty}^{\theta_{0}^{*}}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
&=\left(1-\frac{1+(1-T) r}{1+r}\right) \\
& \times\left(Y-\int_{\theta_{0}^{*}}^{\infty}\left[\int _ { - \frac { M + A - L } { Y } } ^ { Y } \left(M+r-\frac{M+A-L}{Y}\right.\right.\right. \\
&-\int_{-\infty}^{\theta_{0}^{*}}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
&=\left(\frac{T r}{1+r}\right) Y(1+r) \int_{\theta_{0}^{*}}^{\infty}\left[\int_{1+r-\frac{M+A-L}{Y}}^{\infty} g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
&-\int_{-\infty}^{\theta_{0}^{*}}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1} \\
&= \operatorname{Tr} Y \mathbb{P}\left(\theta_{2} \geq 1+r-\frac{M+A-L}{Y}\right) \\
&-\int_{-\infty}^{\theta_{0}^{*}}\left[\int_{-\frac{M+A-L}{Y}}^{\infty}\left(M+A+\theta_{2} Y-L\right) g\left(\theta_{2} \mid \theta_{1}\right) d \theta_{2}\right] f\left(\theta_{1}\right) d \theta_{1}  \tag{3.11}\\
&(3.11) \\
&
\end{align*}
$$

where the penultimate equality follows from (3.7). Observe that the first term of this expression is the same form as debt savings without illiquidity risk (3.6): the tax rate multiplied by the expected interest paid by the bank. However, there is now an extra term representing the counterfactual equity of the bank that is lost due to illiquidity risk in the interim period $\left(\theta_{1}<\theta_{0}^{*}\right)$ when it would have been solvent in the ex-post period $\left(\theta_{2} \geq-\frac{M+A-L}{Y}\right)$. This expression therefore demonstrates the trade-off theory
of financial structure: taxes and illiquidity risk affect the desirability of debt financing in opposite ways.

At this point it is not possible to conclude that adding illiquidity risk always decreases debt savings, because the interest rate $r$ is not the same in (3.6) and (3.11). However, it is reasonable that debt savings will sometimes become negative with the introduction of illiquidity risk. The intuitive explanation is as follows: the existence of a run point $\theta_{0}^{*}$ "zeros out" buyer payoffs whenever $\theta_{1}$ falls below $\theta_{0}^{*}$ in the interim period, because the bank fails due to illiquidity. This has the effect of decreasing buyer payoffs. Because debt buyer payoffs are strictly increasing in $r$, which can be visually verified from the graph in Figure 2.1, if other parameters are held constant, then $r$ must increase to compensate for the decrease in buyer payoffs. Thus, if the run point $\theta_{0}^{*}$ is high enough, the cost of debt $r$ will also be high enough to reduce debt savings over equity, possibly to the extent that they become negative and lead the bank to prefer equity over debt. In effect, illiquidity risk cancels out the tax advantage of debt.

## Chapter 4

## Numeric Analysis

Recall that my goal is to analyze the conditions under which debt is preferred to equity. Specifically, my hypothesis is that because of taxes, the bank prefers debt to equity when there is no illiquidity risk, so debt savings are positive. However, when illiquidity risk is introduced, it increases the cost of debt and outweighs the additional tax benefit from higher interest payments, so debt savings decrease.

Verification of this hypothesis, as well as comparative statics analysis on the exogenous parameters, depends on first deriving the cost of debt $r$ with and without illiquidity risk and then using $r$ to examine debt savings with and without illiquidity risk. In the previous chapter, I derived implicit expressions for $r$ in equations (3.1) and (3.7), and expressions for debt savings in terms of $r$ in expressions (3.6) and (3.11). To derive closed-form expressions for the interest rate $r$ and the debt savings, it is necessary to assume a specific distribution for $f\left(\theta_{1}\right)$ and $g\left(\theta_{2} \mid \theta_{1}\right)$, evaluate the integrals to solve (3.1) and (3.7), and plug the resulting expression for $r$ into (3.6) and (3.11). Although this derivation is theoretically possible, due to the complexity of these expressions, it is practically intractable. If I assume a uniform distribution for $\theta$ as in Morris and Shin, algebraically solving for closed-form expressions for $r$ in order to examine debt savings is quite complicated due to the bounded support of
the uniform distribution. The domain of integration must be split depending on the values of parameters $\theta_{0}, \sigma_{1}, \sigma_{2}$ defining the distribution of the asset return as they relate to the solvency and run points defined by the other exogenous parameters $M$, $A, L, Y, r^{*}$. This leads to complicated casework and any closed-form expressions that result would not be practically meaningful, especially for comparative statics analysis. If I assume another distribution for $\theta$ supported on the entire real line, the complexity of the integrals would also preclude a useful closed-form solution.

Therefore, to analyze my model, I turn to numeric analysis. While this is not quite theoretically complete, at least it's clean.

### 4.1 Methodology

I use MATLAB to numerically solve equations (3.1) and (3.7) for $r$ and then use expressions (3.6) and (3.11) to compute debt savings.

I assume a normal distribution for $\theta$, since it is supported on the entire real line and thus avoids casework due to the boundary issues mentioned above $\prod^{\top}$ Specifically, let

$$
\begin{aligned}
& \theta_{1}=\theta_{0}+\sigma_{1} \varepsilon_{1} \\
& \theta_{2}=\theta_{1}+\sigma_{2} \varepsilon_{2}
\end{aligned}
$$

[^10]where $\varepsilon_{1}, \varepsilon_{2}$ are independently and normally distributed with mean 0 and variance 1 . These expressions imply that ${ }^{2}$
\[

$$
\begin{aligned}
\theta_{1} & \sim N\left(\theta_{0}, \sigma_{1}^{2}\right) \\
\theta_{2} \mid \theta_{1} & \sim N\left(\theta_{1}, \sigma_{2}^{2}\right)
\end{aligned}
$$
\]

The exogenous parameters of the model are as follows:

- Balance sheet: $M, A, L, Y$
- Asset return: $\theta_{0}, \sigma_{1}, \sigma_{2}$
- Outside option: $r^{*}$
- Tax rate: $T$

There are two endogenous parameters representing the interest rate $r$ and the run point $\theta_{0}^{*}$, which depend on each other. $r$ is determined by equations (3.1) and (3.7), and $\theta_{0}^{*}$ is determined by equation (2.7), subject to (2.8).

I assume the following initial parameter values: $M=0.5, A=3, L=2, Y=1$, $\theta_{0}=1, \sigma_{1}=1, \sigma_{2}=1, r^{*}=0.02, T=0.3$. These parameter values imply that the bank initially has the balance sheet shown in Table 4.1. The bank starts with a balance sheet of size 3.5, one-seventh of its assets are liquid, and it is financed with a mixture of long-term debt and equity. The bank then seeks to finance a risky investment of size 1. The resulting liquidity ratio under short-term debt financing is $\frac{M}{Y}=0.5$.

[^11]Table 4.1: Initial Balance Sheet

| Assets | Liabilities \& Equity |
| :--- | :--- |
| Cash: 0.5 | Long-term debt: 2 |
| Non-liquid asset: 3 | Equity: 1.5 |
| Risky investment: 1 | New financing: 1 |

Also, its risky investment is expected to break even since $\theta_{0}=1$, and its standard deviation is 1 in both the interim and ex-post periods. Finally, the outside option provides a return of $2 \%$ and the tax rate is $30 \%$.

The initial interest rate is $r=9.787 \%$ without illiquidity risk and $11.783 \%$ with illiquidity risk. The solvency point is $-\frac{M+A-L}{Y}=-1.5$, the repayment point with illiquidity risk is $1+r-\frac{M+A-L}{Y}=-0.382$, and the run point is $\theta_{0}^{*}=-0.796$. Debt savings are 0.0246 without illiquidity risk and 0.0077 with illiquidity risk.

Since the bank has at least some degree of control over the balance sheet parameters as well as the asset return parameters of the model, and $r^{*}$ intuitively determines the severity of the illiquidity risk, I focus on the exogenous parameters $M, A, L, Y, \theta_{0}$, $\sigma_{1}, \sigma_{2}, r^{*}$ for my comparative statics analysis. ${ }^{3}$. I perturb each exogenous parameter around its initial value and examine the resulting changes in the interest rate. Solving for $r$ using equations (3.1) and (3.7), I obtain graphs of the interest rate and debt savings under various parameter perturbations. In all the interest rate graphs that follow, the solid blue curves represent the interest rate in the absence of illiquidity risk calculated via (3.1), while the dashed red curves represent the interest rate in the presence of illiquidity risk calculated via (3.7). In all of the debt savings graphs that follow, the solid black line represents the debt savings in the absence of illiquidity risk and taxes, calculated via with $T=0$; the dotted blue curve represents the debt savings with taxes alone and no illiquidity risk, calculated via with $T=0.3$; the dashed red curve represents the debt savings with illiquidity risk alone and no taxes,

[^12]calculated via (3.11) with $T=0$; and the dash-dot green curve represents the debt savings with both illiquidity risk and taxes, calculated via (3.11) with $T=0.3$.

### 4.2 Results

### 4.2.1 General Comments

First, some general comments are in order before analyzing the individual parameters. Note that interest compensates debt buyers for both the insolvency risk and illiquidity risk due to the riskiness of the investment $Y$. Thus the interest rate is always positive, and strictly higher with illiquidity risk than without; this compensates debt buyers for the extra condition that the bank be liquid in the interim period in order to receive their payoff.

Nevertheless, observe that since $r^{*}$ only affects the interest rate with illiquidity risk, it can be used as a tuning parameter to weigh the importance of illiquidity risk relative to the tax benefit in determining debt savings. That is, one can adjust the attractiveness of the outside option to adjust the severity of illiquidity risk. The same holds for $T$ : since it only affects the tax benefit of debt and not the illiquidity risk or the interest rate, it can be used as a tuning parameter to weigh the importance of the tax benefit relative to illiquidity risk in determining debt savings. Thus, although the effect of illiquidity risk on the interest rate is clear, the relative importance for debt savings of illiquidity risk and the tax benefit depends on the values of the parameters $r^{*}$ and $T$.

Also observe that for all parameters, the debt savings in the absence of both taxes and illiquidity risk is zero, as shown by the black line in each graph. This is the conclusion of Modigliani and Miller (1958): the bank is indifferent between debt and equity if there is no advantage (taxes) or disadvantage (illiquidity risk) of debt compared to equity. Individually, taxes yield a benefit for the bank, whereas
illiquidity risk yields a loss. In addition, the effects on debt savings of taxes alone and of illiquidity risk alone are not simply additive - that is, the dash-dot green curve is not just the sum of the dashed red and dotted blue curves-because the savings from taxes is calculated at the interest rate in the absence of illiquidity risk, which is lower than the interest rate in the presence of illiquidity risk.

In the discussion that follows, I will repeatedly refer to the solvency point, the minimum value of $\theta_{2}$ such that the bank is solvent:

$$
-\frac{M+A-L}{Y}
$$

and the run point, the minimum value of $\theta_{1}$ such that the bank survives a run:

$$
\theta_{0}^{*}=\left(1+r-\frac{M+A-L}{Y}\right)-\sigma_{2} \Phi^{-1}\left(1-\frac{r^{*} / r}{M / Y}\right)
$$

as well as the "run point condition" required for the run point to be well-defined:

$$
r \geq \frac{r^{*}}{M / Y}
$$

### 4.2.2 Cash $M$



The interest rate graph shows that $r$ decreases with cash $M$. This is sensible because increasing cash decreases the solvency point $-\frac{M+A-L}{Y}$ and thus decreases insolvency risk. It also decreases the run point and thus decreases illiquidity risk as well. Intuitively, increasing the liquidity and assets of the bank should make it less susceptible to
runs and insolvency. In addition, observe that the interest rate increases steeply as $M$ becomes very small. This results from the asymptotic behavior of the run point: as $M$ becomes very small, the $-\sigma_{2} \Phi^{-1}\left(1-\frac{r^{*} / r}{M / Y}\right)$ term and thus the run point increases rapidly, as verified numerically. Intuitively, once liquidity reaches dangerously low levels, debt buyers become fearful of the financial stability of the bank and the risk of runs increases dramatically. Later, it will become clear that liquid cash assets $M$ play a significant role in current policy discussion on reducing the risk of bank runs and improving the stability of the financial system.

The debt savings graph shows that the tax benefit of debt is decreasing with $M$ because it is tied to the interest rate that the bank pays to debt buyers, which is also decreasing with $M$. The loss from illiquidity risk is also decreasing in magnitude with $M$, which is again sensible since adding cash directly improves the liquidity situation of the bank. Finally, the sharp increase in the loss from illiquidity risk at low levels of $M$ mirrors the sharp increase in the interest rate: once liquidity reaches dangerously low levels, the risk of runs increases dramatically. This causes debt savings to become sharply negative; with insufficient liquidity, illiquidity risk makes short-term debt prohibitively expensive, so the bank prefers equity instead.

### 4.2.3 Non-Liquid Assets $A$ and Long-Term Debt $L$




Observe that $A$ and $L$ enter into equations (3.1), (3.6), (3.7), and (3.11) only via their difference $A-L$, so I analyze these parameters together; the effects on the interest rate and debt savings of one is simply the negative of the effects of the other.

The interest rate graph shows that $r$ decreases with non-liquid assets $A$ and increases with long-term debt $L$. This is sensible because increasing "net assets" $A-L$ decreases the chances of insolvency and illiquidity by decreasing the solvency point
and the run point, so a lower interest rate is needed to compensate debt buyers. In addition, observe that the interest rate increases steeply as $A-L$ becomes very small both with and without illiquidity risk. Intuitively, at low levels of net assets, the bank is in greater danger of becoming insolvent or illiquid due to insufficient assets or too much long-term debt, so the amount of interest becomes very sensitive to perturbations in the balance sheet. In fact, as $A-L$ decreases toward a critical value near -0.5 , the interest rate increases asymptotically. This implies that if there are insufficient net assets, the probability of the bank becoming insolvent or illiquid is so high that there is no value of $r$ that can sufficiently compensate debt buyers for taking on the insolvency and illiquidity risks. This has implications for financial crises: with distressed balance sheets, debt markets may break down because buyers are unwilling to lend due to excessive credit risk.

The debt savings graph shows that the tax benefit of debt decreases with nonliquid assets $A$ and increases with long-term debt $L$ when net assets $A-L$ are high. This effect agrees with the effects of $A$ and $L$ on the interest rate, which is sensible since the tax benefit is tied to the interest rate that the bank pays to debt buyers. However, when net assets are low, the tax benefit starts to taper off as $A$ decreases and $L$ increases, even as the interest rate rises steeply. This effect indicates that even though the tax benefit may be larger whenever the bank pays off the debt buyers, the tax benefit is not earned with sufficient probability to realize the larger benefit. After all, the bank must be able to pay off the debt buyers in the first place - that is, $\theta_{2} \geq 1+r-\frac{M+A-L}{Y}$-in order to earn any tax benefit.

At low levels of net assets $A-L$, the loss from the illiquidity risk of debt increases in magnitude as net assets $A-L$ decreases, since the run point increases and the bank is simply more likely to be illiquid. Eventually, at sufficiently low levels of $A-L$, the combination of a lower tax benefit and a greater illiquidity loss makes debt savings negative. Interestingly, at high levels of net assets $A-L$, the loss from illiquidity
risk increases in magnitude as net assets $A-L$ increase. This effect is explained by referring to the second term in the expression for debt savings under illiquidity risk (3.11) when $T=0$ : although the run point is decreasing, the solvency point is also decreasing, thus increasing the loss in the counterfactual equity of the bank due to illiquidity risk when the bank would have been solvent. Since the tax benefit diminishes with a lower interest rate, this loss from illiquidity risk also makes debt savings negative when $A-L$ is too high.

I conclude that the bank usually prefers debt over equity, but with sufficiently extreme values of $A-L$, the bank prefers equity over debt. This occurs either because $A-L$ is too low and the bank is both too likely to become insolvent and too likely to become illiquid, so the illiquidity loss of debt overcomes the tax savings of debt; or because $A-L$ is too high and places too much weight on the loss of counterfactual equity from illiquidity when the bank would have been solvent. Finally, observe that equity preference for low values of $A-L$ only occurs at very high interest rates-near $100 \%$. Though this may simply be a consequence of the specific parameters used in this numerical analysis, the high interest rate and unattractiveness of debt at low levels of $A-L$ emphasizes how distressed balance sheets can spell trouble for banks seeking financing.

### 4.2.4 Investment $Y$



The interest rate graph shows that $r$ increases with the investment amount $Y$. This is sensible because increasing the investment amount increases the chance of insolvency and illiquidity by increasing the solvency point and the run point, so a higher interest rate is needed to compensate debt buyers. Without illiquidity risk, the effect on $r$ of
increasing $Y$ gradually tapers as $Y$ increases because of the functional form of the solvency point. With illiquidity risk, the interest rate begins to increase asymptotically as $Y$ becomes large, due to the asymptotic behavior of the run point: as $Y$ becomes very large, the $-\sigma_{2} \Phi^{-1}\left(1-\frac{r^{*} / r}{M / Y}\right)$ term becomes very large and thus the run point increases rapidly, as verified numerically. Intuitively, once the required investment amount $Y$ becomes very large, the risk of illiquidity becomes too great and there is no value of $r$ that can sufficiently compensate debt buyers. Note that this effect is only present with illiquidity risk: the solvency point is not sensitive to $Y$ at high levels of $Y$, so insolvency risk by itself has little consequence.

The debt savings graph shows that the tax benefit of debt increases with $Y$ because it is tied to the interest rate that the bank pays to debt buyers, which is also increasing in $Y$. The illiquidity loss of debt dramatically increases in magnitude since the run point, and therefore the probability of illiquidity, is dramatically increasing in $Y$. At low levels of $Y$, the interest rate is so low that the tax benefit does not outweigh the illiquidity loss, though both are quite small. At moderate values of $Y$, the interest rate increases enough for the tax benefit to weakly dominate, and at high values of $Y$, illiquidity risk quickly overwhelms the tax benefit. I conclude that if the investment amount $Y$ is low enough, then the bank somewhat prefers short-term debt, but if the investment amount is too high, the bank definitely prefers equity due to significant illiquidity risk.

### 4.2.5 Ex-Ante Expected Asset Return $\theta_{0}$



Observe that the graphs for $\theta_{0}$ are almost identical to the graphs for $A$, save for changes in scale. The reasoning is also similar. This suggests that the effect of having a higher expected asset return is similar to the effect of having a larger amount of assets altogether.

The interest rate graph shows that the interest rate $r$ decreases with the ex-ante expected asset return $\theta_{0}$. This is sensible because increasing $\theta_{0}$ directly decreases the chances of insolvency and illiquidity since $\theta_{2}$ and $\theta_{1}$ are less likely to fall below the solvency and run points, respectively. Thus a lower interest rate is needed to compensate debt buyers. In addition, the interest rate increases asymptotically as $\theta_{0}$ decreases toward a critical value near -0.5 , both with and without illiquidity risk. This is because the probability of the bank becoming insolvent or illiquid is so high at such an extreme value of $\theta_{0}$ that there is no value of $r$ that can sufficiently compensate debt buyers for taking on the risk. Again, this has implications for financial crises: if the assets on a bank's balance sheet are suddenly perceived to have a lower future value than before, debt markets may break down because buyers are unwilling to lend due to excessive credit risk.

The debt savings graph shows that the tax benefit of debt decreases with $\theta_{0}$ when $\theta_{0}$ is high. This effect is the same as the effect of $\theta_{0}$ on the interest rate, which is sensible since the tax benefit is directly tied to the interest paid by the bank. However, when $\theta_{0}$ is low, the tax benefit starts to taper off $\theta_{0}$ decreases, even as the interest rate rises steeply. This effect indicates that even though the tax benefit may be larger whenever the bank pays off the debt buyers, the tax benefit is not earned with sufficient probability to realize the larger benefit because $\theta_{2}$ is expected to be below the solvency or illiquidity points too often.

The loss from the illiquidity risk of debt also increases in magnitude as $\theta_{0}$ decreases, since the bank is simply more likely to be illiquid. Eventually, at sufficiently low levels of $\theta_{0}$, the combination of a lower tax benefit and a greater illiquidity loss actually makes debt savings negative. Interestingly, at high levels of $\theta_{0}$, the loss from illiquidity risk increases in magnitude as $\theta_{0}$ increases. This effect is explained by referring to the second term in the expression for debt savings under illiquidity risk (3.11) when $T=0$ : although the bank is less likely to be illiquid as $\theta_{0}$ increases, it experiences a
greater loss in the counterfactual equity of the bank due to illiquidity risk when the bank would have been solvent. Since the tax benefit diminishes with a lower interest rate, this loss from illiquidity risk also makes debt savings negative when $A-L$ is too high.

I conclude that the bank usually prefers debt over equity, but with sufficiently extreme values of $\theta_{0}$, the bank prefers equity over debt. This occurs either because $\theta_{0}$ is too low and the bank is both too likely to become insolvent and too likely to become illiquid, so the illiquidity loss of debt overcomes the tax savings of debt; or because $\theta_{0}$ is too high and places too much weight on the loss of counterfactual equity from illiquidity when the bank would have been solvent. Finally, observe that equity preference for low values of $\theta_{0}$ only occurs at very high interest rates-near $100 \%$. Though this may simply be a consequence of the specific parameters used in this numerical analysis, the high interest rate and unattractiveness of debt at low levels of $\theta_{0}$ emphasizes how unfavorable prospects for a bank's assets can spell trouble.

### 4.2.6 Volatility of Asset Return $\sigma_{1}, \sigma_{2}$





The interest rate graphs show that $r$ is increasing in both the first-period standard deviation of asset return $\sigma_{1}$ and the second-period standard deviation of asset return $\sigma_{2}$. This can be explained by referring to the buyer payoff structure graphed in Figure 2.1. Initially, when $\sigma$ is very low, the probability of illiquidity or insolvency is also very low, since the asset return is almost guaranteed to fall very close to $\theta_{0}=1$. Thus the payoff is likely to be the full amount $Y(1+r)$ and the interest rate is low, since the
debt buyers require less compensation to break even in expectation. As $\sigma$ increases, the probabilities of illiquidity and insolvency increase, since $\theta_{1}$ and $\theta_{2}$ are more likely to fall below the run point and solvency point, respectively. Thus the interest rate is higher when $\sigma$ is higher in order to compensate debt buyers for the added risk.

In addition, observe that the difference in interest rates with and without illiquidity risk is different for $\sigma_{1}$ and $\sigma_{2}$. Since $\sigma_{1}$ does not directly affect the run point except through $r$-once the interim period is reached, $\sigma_{1}$ is no longer relevant-the effect of illiquidity risk in increasing the interest rate is more or less constant for different values of $\sigma_{1}$. However, increasing $\sigma_{2}$ directly decreases the run point as verified numerically, so the effect of illiquidity risk in increasing the interest rate diminishes at larger values of $\sigma_{2}$. Intuitively, if the ex-post asset return $\theta_{2}$ is very uncertain in the interim period, then even if $\theta_{1}$ is very low, there is a good chance that $\theta_{2}$ will still be sufficiently high enough for the bank to be solvent ex-post.

The debt savings graphs show that the tax benefit of debt increases with $\sigma$. This effect is the same as the effect of $\sigma$ on the interest rate, which is sensible since the tax benefit is directly tied to the interest paid by the bank. As previously discussed, the loss from illiquidity risk is relatively constant as $\sigma_{1}$ increases, since the volatility of the asset return in the interim period is irrelevant once the interim period is reached, and the loss from illiquidity risk diminishes to zero as $\sigma_{2}$ increases, since the run point decreases with $\sigma_{2}$. Overall, debt savings are increasing in both $\sigma_{1}$ and $\sigma_{2}$. However, observe that if $\sigma$ is too small, then the interest rate is too low for the tax savings to outweigh the illiquidity loss. I conclude that the bank prefers debt over equity once the volatility of its assets is high enough for the tax benefit to dominate.

### 4.2.7 Outside Option $r^{*}$



The interest rate graph shows that the interest rate and tax benefit in the absence of illiquidity risk do not depend on the value of the outside option $r^{*}$. This is sensible because the parameter $r^{*}$ is specific to illiquidity risk and does not enter into (3.1) or (3.6), the equations defining the interest rate and debt savings in the absence of
illiquidity risk. This confirms the earlier discussion that because $r^{*}$ is independent of the tax benefit of debt, it can be interpreted as a tuning parameter to adjust the significance of illiquidity risk relative to the tax benefit of debt.

In the presence of illiquidity risk, the interest rate $r$ is increasing in the outside option $r^{*}$. This is again sensible because the run point $\theta_{0}^{*}$ is increasing in $r^{*}$ as verified numerically, implying that a higher $r^{*}$ yields a higher probability of illiquidity. Intuitively, as the outside option becomes more attractive to debt buyers, they will be more easily persuaded to run and thus require a higher interest rate to be compensated for this added illiquidity risk.

The debt savings graph shows that the illiquidity loss of debt increases in magnitude as the outside option $r^{*}$ increases. The same explanation holds: as the outside option becomes more attractive to debt buyers, the run point increases as they are more easily persuaded to run, so the bank experiences a greater expected loss from illiquidity risk. This effect overcomes the tax benefit from an increased interest rate. I conclude that the bank prefers debt at lower values for the outside option $r^{*}$ and prefers equity at higher values.

### 4.2.8 Summary

By (3.6), debt savings are always positive in the absence of illiquidity risk due to the tax benefit of debt interest. However, in the presence of illiquidity risk, debt savings becomes negative at low values of $M, \sigma_{1}$, and $\sigma_{2}$; both low and high values of $A, L$, and $\theta_{0}$; and high values of $Y$ and $r^{*}$, leading to a breakdown of the pecking order: debt is no longer preferred to equity.

A key finding of my model is that if the bank has too little liquidity, has too little net assets, has too large of an investment, has too low of an expected asset return, or faces a very attractive outside option, illiquidity risk becomes excessive due to a high run point and thus outweighs the tax benefit. The interest rate with illiquidity
risk also becomes very high. In these cases, investors demand a considerable amount of compensation for taking on illiquidity and insolvency risk and are more enticed to run, due to the weak finances of the bank relative to the outside option. As a result, current policy efforts to keep credit markets functioning during a crisis, reduce the risk of bank runs, and improve the stability of the financial system are particularly concerned with preventing extreme values of these parameters.

A somewhat curious but also significant finding is that too much net assets or too high of an expected asset return also makes debt less attractive, because the loss of counterfactual equity due to illiquidity risk when the bank would have been solvent is weighted very heavily.

Finally, if the bank has too little volatility, the tax savings of debt are not enough to outweigh the illiquidity loss.

It is important to emphasize that insolvency risk alone cannot make debt savings negative, because both debt and equity face the same insolvency risk and debt has the advantage of being tax-deductible. Only when illiquidity risk is added do the debt savings decline, and my model finds that they are erased at certain parameter values due to the threat of bank runs, leading to the bank preferring equity over debt.

## Chapter 5

## Game-Theoretic Extension

The model of financial structure and illiquidity risk that I have solved thus far assumes that the bank always receives the type of financing that it requests from investors in the financial markets, so it is free to choose whichever type of financing-short-term debt or equity - yields a higher payoff. However, Noe (1988) provides an alternative framework for analyzing corporate financial structure, treating the financing process as a sequential signaling game. I can extend my model to incorporate the essential elements of Noe's model, but it is beyond the scope of this senior thesis to solve such an extension completely. Nevertheless, in this chapter I provide an informal discussion of my extended model and qualitative predictions concerning a solution.

### 5.1 Noe Model

In the Noe model, the bank initially has a certain cash flow $t$ generated by existing assets, and would like to make an investment $I$ in the ex-ante period to generate additional cash flow in the ex-post period. The bank must raise funds for the investment from security buyers in the financial markets. However, while $I$ is common knowledge, security buyers do not know $t$, so there is asymmetric information. Security buyers do have a common prior probability distribution $p$ over a finite number of
bank types $\{t\}_{i=1}^{n}$. Noe also considers both cases in which the bank does or does not have uncertainty about its own cash flows $t$.

The bank first requests a type of financing-debt, equity, or nothing. Following this request, the buyers update their beliefs on the possible types of the bank and either provide the requested funding at some terms (an interest rate $r$ for debt; a share $\alpha$ for equity) or reject the request. If funding is provided, the buyers expect to break even by Bertrand-like competition.

The result of Noe's analysis is that under certain conditions, if the bank faces no uncertainty about its cash flows, then all bank types weakly prefer debt to equity. However, if the bank does face uncertainty about its cash flows, then some bank types may strictly prefer equity. Noe argues that this is a breakdown of the "pecking order" theory of corporate finance explained in Section 1.2 .

### 5.2 Adaptation of the Noe Model

While Noe's model of corporate finance does not account for illiquidity risk, it does provide a framework for incorporating asymmetric information and game theory. I can therefore adopt elements of Noe's model to inform my analysis of corporate finance and illiquidity risk. My analysis in the preceding sections assumes that both the bank and the buyers know all details of the bank's balance sheet and the distribution of the ex-post return on the risky asset $\theta_{2}$. I can therefore interpret my model as a sequential game with complete and perfect information. First, the bank chooses to finance the investment $Y$ with either debt or equity, depending on the analysis of debt savings above. Next, if the bank chooses to invest, the financial markets provide funding and expect to break even due to Bertrand-like competition if the required debt interest $r$ or equity share $\alpha$ exists following the analysis in Chapters 3 and 4 .

Otherwise, the markets can refuse to provide funding if illiquidity risk or insolvency risk is so high such that no interest rate $r$ is high enough to compensate.

Now I assume that there is asymmetric information: the buyers know the balance sheet of the bank, but their knowledge of the distribution of $\theta_{2}$ is limited to its shape only and not its center. In terms of my model, all exogenous parameters except for $\theta_{0}$ are common knowledge to the bank and security buyers, but $\theta_{0}$ is only known to the bank. For example, if the bank knows that $\theta_{2}$ is distributed uniformly on $\left[\theta_{0}-\frac{\sigma}{2}, \theta_{0}+\frac{\sigma}{2}\right]$, then the buyers know that $\theta_{2}$ is distributed uniformly on an interval of length $\sigma$, but not its mean $\theta_{0}$. In game-theoretic terms, from the buyers' perspective the bank is one of $N$ types $\theta_{0}^{1}, \theta_{0}^{2}, \ldots, \theta_{0}^{N}$. The buyers do not know the bank's type, but have some prior belief given by a probability distribution $p$ over the types. The return of the bank's risky asset is given by a distribution $F_{\theta_{0}}$ parameterized by the bank's type $\theta_{0}$, which represents the mean of the distribution.

Introducing asymmetric information on the $\theta_{0}$ parameter in this way is a natural way to incorporate the uncertainty of cash flows that is present in Noe's model. Both the bank and the security buyers are uncertain about the eventual return of the risky asset $Y$ due to the fact that $\theta_{2}$ has a probability distribution with positive standard deviation parameters $\sigma_{1}$ and $\sigma_{2}$. In the language of Noe's model, this corresponds to the "residual uncertainty" $Z$ faced by insiders at the bank. However, the bank knows the ex-ante expected return of the risky asset $\theta_{0}$, whereas the buyers have additional uncertainty about the cash flows because they do not know $\theta_{0}$.

### 5.3 Equilibrium

I now turn to a qualitative discussion of the equilibrium of my extended model. Observe that an equilibrium here consists of a set of types of banks that request debt and a set of types that request equity, as well as an interest rate $r$ and a share $\alpha$
which the buyers demand in return for funding the investment. Because of incomplete information, upon receiving the financing request from the bank, the buyers update their beliefs on the type of the bank, as they now know which set (debt-requesting or equity-requesting) the bank belongs to. The buyers set a single interest rate $r$ for all bank types requesting debt financing and a single equity share $\alpha$ for all bank types requesting equity funding. The buyers demand an interest rate $r$ such that on average, over all the bank types that request debt funding, the buyers expect to break even; the same holds for equity shares $\alpha$.

Deriving the solution to my extended model would follow the process used in Chapter 3 with some modification. Because debt buyers do not know $\theta_{0}$, the breakeven conditions (3.1) and (3.7) must contain an extra expectation over bank types in the form of a discrete weighted average over possible ex-ante asset returns $\left\{\theta_{0}^{i}\right\}_{i=1}^{N}$. On the bank's side, for every bank type $\theta_{0}^{i}$ the debt savings expressions 3.6 and (3.11) would be unchanged, but the interest rate $r$ would be the common interest rate demanded by the buyers.

To gain intuition on how the actions of the bank may change under asymmetric information, I now consider the effect of the introduction of asymmetric information on the bank's payoffs. By the results of Chapter 4 and expression (3.9), respectively, the interest rate $r$ and the equity share $\alpha$ are both decreasing in $\theta_{0}$. However, with incomplete information, the buyers must set a common interest rate $r$ or equity share $\alpha$ for all bank types requesting debt or equity financing, respectively. Therefore, a bank with a high type $\theta_{0}^{i}$ will find that with incomplete information, the $r$ or $\alpha$ demanded by the buyers is higher than what would be demanded under complete information, where the buyers can customize the financing terms to each bank type. These bank types will find that their expected payoff decreases under incomplete information. Conversely, a bank type with low type $\theta_{0}^{i}$ will find that with incomplete information, the $r$ or $\alpha$ demanded by the buyers is lower than what would be demanded under
complete information. These bank types will find that their expected payoff increases under incomplete information. Intuitively, banks that are of higher type $\theta_{0}^{i}$ are penalized because security buyers do not know that their type is high, and banks that are of lower type $\theta_{0}^{i}$ benefit from pooling with the high types.

With this intuition, I now characterize a possible equilibrium of the extended model. Recall that the results of Chapter 4 indicated that banks prefer debt to equity financing when $\theta_{0}$ is neither too high nor too low. In other words, there exist two distinct thresholds at which the bank's preference switches between debt and equity. Since banks at the extreme ends of the distribution of $\theta_{0}$ have a strong preference for equity under complete information, and banks close to the middle have a strong preference for debt, it is less likely that their preference will switch between debt and equity under incomplete information. Thus, after introducing incomplete information, it is a reasonable proposition that the equilibrium of my model will still be characterized by two thresholds, between which debt is preferred to equity. However, because higher types are penalized by pooling with lower types, a bank whose type is just below a threshold in the complete information model may find that it is more profitable to switch their preference in the incomplete information model, because it can now pool with higher bank types on the other side of the threshold instead of lower bank types on the same side. Therefore, it is reasonable to expect that these thresholds will be lower with incomplete information.

To summarize, a plausible equilibrium for my extended model is as follows. Debt and equity buyers set a common $r$ and $\alpha$ for all bank types requesting debt or equity financing, respectively, because they do not know the type of bank to which they are offering financing. The $r$ and $\alpha$ are determined in the same manner as in Chapter 3, but with an extra expectation over bank types. The types of banks that request debt financing are located between two thresholds for $\theta_{0}$, as in the model with complete information, but these thresholds are lower because some bank types just below a
threshold will want to pool with higher types above the threshold instead of lower types on the same side of the threshold. Some banks, such as the ones with the lower values of $\theta_{0}$ out of those requesting debt, benefit from financing terms that are more favorable than they would have received under complete information, while others, such as the ones with the higher values of $\theta_{0}$ out of those requesting debt, would prefer the financing terms offered under complete information.

### 5.4 Consequences for Comparative Statics

To conclude the qualitative discussion of my extended model, I consider the comparative static effects of the exogenous parameters, which were analyzed under complete information in Chapter 4. The key consequence of this game-theoretic extension is to introduce asymmetric information in the form of the distribution of the asset return $\theta$. Therefore, since the other exogenous parameters $M, A, L, Y, \sigma_{1}, \sigma_{2}$, and $r^{*}$ are common knowledge to the bank and to security buyers, the results of the comparative statics analysis in Chapter 4 should still hold for these parameters.

However, changes in $\theta_{0}$ are now not directly observable by security buyers. This implies that an exogenous change in the true $\theta_{0}$ does not change the prior beliefs $p$ of the security buyers. In equilibrium, because of pooling, an exogenous change in $\theta_{0}$ for a bank will not change the interest rate or equity share at which it finances its investment (unless the bank changes financing types) because security buyers do not observe the change in $\theta_{0}$. Therefore, to examine the comparative statics effect of $\theta_{0}$, I fix $r$ and examine the debt savings (3.11) as a function of $\theta_{0}$ only.

Figure 5.1 graphs debt savings as a function of $\theta_{0}$ in the same fashion as the graphs in Chapter 4, but with a fixed $r$ 卫 It is evident that when security buyers cannot observe changes in $\theta_{0}$, an exogenous increase in $\theta_{0}$ therefore does not change $r$ and

[^13]strictly increases debt savings. The conclusion differs from that of Chapter 4, since here debt savings are always increasing in $\theta_{0}$, even at high values of $\theta_{0}$. Intuitively, the loss of the bank's counterfactual equity when it is illiquid but still solvent is less significant, and at high levels of $\theta_{0}$, the fixed interest rate $r$ is higher than it would have been under complete information and thus increases the tax benefit. I conclude that under asymmetric information where the bank has more information than security buyers, due to the pooling of financing terms, the comparative statics effect of increasing $\theta_{0}$ is to increase debt savings at all levels of $\theta_{0}$.

Figure 5.1: Debt Savings as a Function of $\theta_{0}$, Fixed $r$


## Chapter 6

## Policy Implications

Recall that the goal of this paper is to analyze the preference of a bank between issuing short-term debt or equity. The fundamental trade-off is that short-term debt is cheaper due to tax incentives but also more expensive due to illiquidity risk, whereas equity is not subject to either tax benefits or illiquidity risk.

However, there is an externality that I have not considered in my model. When the bank becomes illiquid, its counterparties (other than its short-term debt buyers) may suffer losses. These counterparties are also often banks, which can in turn also become illiquid and fail if their creditors decide to run. Illiquidity risk therefore poses a risk to the stability of the financial system as a whole due to the interconnectedness of financial institutions. As a result, and especially after this systemic risk played a major role in the financial crisis of 2007-2008, policymakers and regulators have sought ways to reduce illiquidity risk in order to improve the health of the financial system and prevent such crises in the future.

### 6.1 Mitigating Illiquidity Risk

There are several ways in which regulators and banks can reduce illiquidity risk 1

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### 6.1.1 Increasing Cash

One straightforward method to alleviate illiquidity risk is for regulators to mandate that banks hold more liquid cash reserves to cover short-term debt withdrawals. If short-term debt buyers observe that the bank is quite liquid, they will be less likely to run. In my model, this corresponds to increasing cash $M$ relative to short-term debt financing $Y$. This lowers both the solvency point and the run point, so the bank is more likely to be liquid in the interim period and solvent in the ex-post period. However, though not present in my model, holding cash instead of non-liquid assets comes at a cost to the bank because non-liquid assets with longer maturities generally yield higher returns to the bank - a consequence of the normally upward-sloping yield curve, as described in the next section.

### 6.1.2 Reducing Short-Term Debt

Conversely, instead of increasing cash holdings, another method to reduce illiquidity risk is to decrease the short-term debt holdings of banks, since only short-term debt carries illiquidity risk. To replace short-term debt financing, banks may turn to equity financing, which is the trade-off I examine in this paper. However, from the examination of the difference in the bank's payoff under short-term debt and under equity financing, this is costly for the bank whenever debt savings are positive.

Banks could also issue long-term debt $L$ instead of short-term debt. However, short-term debt is generally cheaper than long-term debt, ceteris paribus. This fact is captured in the usually upwards-sloping yield curve: for a set of debt instruments differing only in their maturity date, instruments with longer maturities yield higher interest rates. This extra compensation demanded by debt buyers for longer maturities can be explained by a variety of factors, including the fact that investors demand a premium for a longer period of exposure to interest rate movements and the possibility of default.

An extension of my model to include long-term debt as an option for the bank can qualitatively be described as follows. The interest rate promised by the bank on long-term debt, instead of being determined by a break-even condition for the buyers, would be exogenously higher than the interest rate on short-term debt due to the term structure of interest rates as described above - perhaps the short-term debt rate plus a "maturity premium" that can vary depending on the other exogenous parameters. Relative to short-term debt financing, the bank faces an increased repayment cost with long-term debt financing, but this is at least partially offset by a greater tax benefit from the higher interest rate and the lack of illiquidity risk associated with long-term debt financing. Essentially, by switching from short-term to long-term debt, the bank trades away illiquidity risk for greater tax savings but also greater insolvency risk, due to a higher solvency point resulting from the higher interest rate. Relative to equity financing, long-term debt still carries a tax benefit, but due to a higher interest rate it is unclear which is more profitable for the bank in expectation. It remains to compare the bank's payoffs under all three types of financing-short-term debt, long-term debt, and equity - to observe which one is preferred. Of course, the answer will change depending on the values of exogenous parameters. Qualitatively, long-term debt will do better than short-term debt when illiquidity risk is significant or tax rates are high, and do better than equity when insolvency risk is low or tax rates are high. Of course, banks will suffer a loss if regulators mandate a reduction of short-term debt when it yields the highest payoff to banks.

### 6.1.3 Central Bank Intervention

Alternatively, instead of regulating bank balance sheets, policymakers can also seek to simply alleviate the effects of illiquidity risk. For example, the central bank can offer emergency funding to banks suffering from a bank run in an arrangement similar to the deposit insurance advocated by Diamond and Dybvig (1983). Short-term
debt holders would receive their promised payoff and the bank would not fail from temporary illiquidity when it would otherwise be solvent. This would eliminate the loss appearing in the second term of short-term debt savings (3.11). However, such a "bailout" from the central bank comes with its own practical, political, and moral hazard risks. Both banks and investors may engage in excessive risk-taking, knowing that the central bank or the government will step in to cover any resulting losses.

### 6.2 Basel III Regulatory Framework

I now examine the policies that are currently in place to reduce illiquidity risk and discuss the policy implications of my model.

The Basel III regulatory framework includes quantitative standards for regulating liquidity ${ }^{2}$ Established in 2010-2011 in the wake of the financial crisis of 2007-2008, it imposes capital and liquidity requirements with the goal of avoiding another disastrous crisis. The liquidity requirements of Basel III consist of two ratios: the liquidity coverage ratio and the net stable funding ratio.

### 6.2.1 Liquidity Coverage Ratio

The liquidity coverage ratio (LCR) is defined as the ratio of high quality liquid assets to projected cash claims over the next 30 days, which are calculated by multiplying short-term liability balances by runoff rates. The goal is to ensure that banks hold enough cash to survive a 30-day market crisis, after which central banks and governments will have had enough time to take emergency measures. Basel III requires that the LCR be at least 1 .

As described in Chapter 2, whenever the bank finances with short-term debt, the liquidity coverage ratio corresponds to $\lambda=\frac{M}{Y}$, the portion of short-term debt $Y$ that

[^15]can be repaid with (is "covered" by) liquid cash assets $M \sqrt{3}^{3}$ Observe that the run point (2.7) is unambiguously decreasing in the liquidity coverage ratio $\lambda=\frac{M}{Y}$. Therefore, taking the other parameters as given, requiring a minimum value of $\frac{M}{Y}$ puts an upper bound on the run point $\theta_{0}^{*}$, which is useful in reducing the probability that the bank is illiquid in the interim period. However, requiring a minimum LCR is not a cure-all for reducing illiquidity risk because there still remain degrees of freedom from the other exogenous parameters. For example, the run point is decreasing in net assets $A-L$, so a low value of $\frac{M}{Y}$ can still result in a high run point if the bank has very little net assets. Also, a low value of $\theta_{0}$ can result in high levels of illiquidity risk, even with a low run point, if $\theta_{1}$ is very likely to fall below the run point.

The results from Chapter 4 indicate that due to the losses from illiquidity risk dominating the tax benefit of short-term debt, the interest rate $r$ is decreasing in $M$ and increasing in $Y$, and debt savings are increasing in $M$ and decreasing in $Y$. Therefore, if the bank increases its liquidity coverage ratio $\frac{M}{Y}$ by changing $M$ or $Y$ while holding the other fixed, the interest rate $r$ decreases and debt savings increases.

Based on the results of my model, the LCR requirement of Basel III therefore implements the solutions of Section 6.1 to reduce illiquidity risk: increasing cash, decreasing short-term debt, and implicitly calling for central bank intervention via the 30-day standard for measuring short-term debt liabilities. In doing so, the LCR requirement increases the savings of short-term debt financing over equity, because the short-term debt is now somewhat "safer" due to increased liquidity and a decreased run point. However, it is not a comprehensive solution, since there are other

[^16]parameters such as net assets that also affect illiquidity risk. In addition, mandating that a certain portion of assets be liquid is costly for the bank, since liquid assets pay less than non-liquid assets. The reduction in liquidity risk due to the LCR must therefore be weighed against the loss in payoffs to banks.

Finally, there is the question of calibration: is an LCR requirement of 1 a sensible choice? While a high liquidity requirement reduces the risk of illiquidity crises, it also imposes real costs on the economy by forcing banks to hold cash that could otherwise be used to perform their normal business in making loans and investments (Elliott 2014, 6-8). Also, note that banks will usually keep a liquidity "buffer" above the requirement, so that they are less likely to fall below the requirement and trigger adverse regulatory or market responses, especially in times of crisis (Elliott 2014, 613). In my model, an LCR of at least 1 implies that runs are not possible, because short-term debt holders can always be repaid in full if they run. The risk of illiquidity is thus removed entirely, which seems to be a dramatic consequence. This raises the concern that the LCR may not adequately account for the trade-offs involved in mandating that banks hold a large amount of cash. However, given practical concerns such as debt buyers' imperfect knowledge of the bank's balance sheet, setting an LCR of 1 may not be too high after all, because the extra liquidity held by the bank helps to assuage debt buyers' concerns concerning the strength of the bank's finances.

### 6.2.2 Net Stable Funding Ratio

The net stable funding ratio (NSFR) is defined as the ratio of available stable funding (i.e. equity and long-term debt, but not short-term debt) to required stable funding (as determined by the liquidity and maturity characteristics of assets), over the time horizon of one year. The goal is to ensure that risky, non-liquid assets are adequately supported by stable funding sources, discouraging the excessive use of short-term debt. Basel III requires that the NSFR be at least 1.

In my model, the NSFR roughly corresponds to the ratio $\frac{L+E}{Y+A}$ : the stable funding sources of long-term debt and equity support the non-liquid assets $A$ and $Y$. Since the balance sheet must balance before the investment, it follows that $E=M+A-L$ and thus the NSFR is equal to $\frac{M+A}{Y+A}$. Observe that $\frac{M}{Y} \geq 1 \Longleftrightarrow \frac{M+A}{Y+A} \geq 1$; thus the LCR is at least 1 if and only if the NSFR is at least 1 . This degenerate relationship between the LCR and NSFR counterparts in my model is a consequence of the simplified balance sheet used in my model; it ignores complicated asset classes and abstracts away from the details of the calculation of the two ratios. Nevertheless, for the purposes of providing insight into the policy implications of my model, the discussion of the LCR applies here as well.

## Chapter 7

## Conclusion

Illiquidity risk and financial structure are two areas of active research, but the existing literature does not examine them together. Following the framework for credit risk in Morris and Shin (2016), this paper constructs a model for the financial structure decision of a bank in light of illiquidity and insolvency risk. Key aspects of the model include a threshold-type run point defining a level of asset return above which all short-term debt holders roll over and below which all short-term debt holders run, as well as the assumption that both short-term debt holders and equity holders expect to break even due to competitive financing markets.

An implicit analytic solution to the model is readily obtained for general distributions of asset returns, but closed-form solutions for the interest rate and debt savings are considerably more complicated. Therefore, numeric analysis is useful for performing comparative statics analysis. I find that the tax benefit of debt can be outweighed by the negative effects of illiquidity risk due to a high run point when liquidity, net assets, or the expected asset return is too low; or when the investment amount or the outside option return is too high. The interest rate is also very high under these extreme parameter values, which correspond to a bank with a distressed balance sheet finding it difficult to raise funding from investors in the financial mar-
kets. In addition, high amounts of net assets or high levels of expected asset return can also lead to a preference for equity because a large loss of payoff results when the bank is illiquid but would have been solvent. Finally, if asset volatility is too low, the tax benefit of debt cannot overcome the loss from illiquidity risk. The existence of conditions under which equity is preferred to debt constitutes a breakdown of the pecking order theory of financial structure.

I then qualitatively discuss an extension of my model to a sequential signaling game framework, similar to that of Noe (1988). Introducing asymmetric information by making $\theta_{0}$ known to insiders only, I reason that the conclusions concerning when debt is preferred to equity should generally be preserved, while the comparative statics analysis of $\theta_{0}$ now predicts that debt savings are always increasing in the expected asset return $\theta_{0}$, instead of decreasing at high levels of $\theta_{0}$.

The analysis of illiquidity risk and financial structure has important policy implications. The recent liquidity coverage ratio (LCR) requirement of the Basel III regulatory system mandates that the ratio of liquid assets to short-term debt liabilities be at least 1. I find that setting a minimum LCR puts an upper bound on the run point, which is useful in reducing the probability that the bank is illiquid in the interim period. However, a comprehensive regulatory approach would also need to consider the other exogenous parameters, such as net assets of the bank, that can also affect the probability of illiquidity.

While my model presents a useful framework for studying both the illiquidity risk and financial structure of banks, it is only a starting point. Further research is needed to complete the analysis of Chapter 5 under asymmetric information in a strategic context. With sufficient conditions on the distributions of the asset return, perhaps tractable closed-form solutions for the interest rate and the debt savings can be found, which may yield useful insights and more general comparative statics results. Nevertheless, it is clear that endogenizing the bank's financial structure provides a
new dimension for the study of illiquidity risk, bank runs, and financial crises. With this added insight, policymakers and industry leaders alike can promote a healthy financial system and global economic stability.

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## PLEDGE:

This paper represents my own work in accordance with University regulations.



[^0]:    ${ }^{1}$ Of course, fractional-reserve banking is not inherently bad, as it is the mechanism by which banks can act as financial intermediaries between borrowers and savers. However, it does carry the risk of bank runs and illiquidity.

[^1]:    ${ }^{2}$ See Gorton (2012) for an excellent overview of financial crises and Gorton (2010) for a specific treatment of the 2007-2008 financial crisis.

[^2]:    ${ }^{3}$ For the remainder of this paper, I assume that the firm in question is a financial institution, called a "bank" for convenience. This assumption has no effect on the analysis, other than to emphasize the fact that illiquidity risk is most relevant when applied to banks and the financial system.

[^3]:    ${ }^{4}$ These differences are elaborated upon in Chapter 2

[^4]:    ${ }^{1}$ Other ratios commonly used to measure financial structure include, assuming $Y$ is financed with debt, the debt-equity ratio $\frac{L+Y}{E}$ and the asset-equity (leverage) ratio $\frac{M+A+Y}{E}$.

[^5]:    ${ }^{2}$ The slight discontinuity in the blue curve at $\theta_{2}=1+r-\frac{M+A-L}{Y}$ in Figure 2.2 is due to the tax benefit of debt: I assume that the debt buyers gain control of the bank if repayment would result in insolvency even if the tax benefit of debt would "save" the bank from insolvency. With this payoff structure, the tax rate then affects only the bank's payoff and not the buyers' payoff or the interest rate $r$.

[^6]:    ${ }^{3}$ See Morris and Shin (2010, 11-14 and 39-41) and Morris and Shin (2016, 10-12). The analysis presented in these papers assumes that $\theta_{1}$ and its noise are uniformly distributed while referring to Morris and Shin (2003) to extend the argument to general densities.

[^7]:    ${ }^{4}$ Later, in my numerical analysis, I use a normal distribution for ease of computation.
    ${ }^{5}$ In addition, Morris and Shin (2010, 8-9) use a parameter $\psi$ to represent the amount of cash that can be raised from a unit of the risky asset in the interim period. I set this parameter equal to 0 by having both non-cash assets $A$ and $Y$ be totally illiquid. Since the parameter is exogenous to the model of Morris and Shin anyway, this assumption is innocuous.

[^8]:    ${ }^{6}$ This derivation also holds for general distributions of $\varepsilon_{2}$ by substituting a general cumulative distribution function $F$ for $\Phi$.
    ${ }^{7}$ See Morris and Shin (2010, 39-41) for their derivation and justification of the run point in a global game context under a general distribution.

[^9]:    ${ }^{8}$ This is a somewhat extreme assumption: essentially, neither party recovers any payoff from the remaining assets of the bank if it becomes illiquid.

[^10]:    ${ }^{1}$ Recall that from the discussion in Section 2.3.1. the validity of the results of Morris and Shin in characterizing illiquidity risk hold for general distributions.

[^11]:    ${ }^{2}$ Note that the asset returns $\theta_{1}$ and $\theta_{2}$ can be negative - that is, the value of the risky asset can become negative in the interim or ex-post periods. This corresponds to states of the world in which the investment opportunity results in a loss. In my model, this is necessary to allow for insolvency risk, since the solvency point $-\frac{M+A-L}{Y}$ is always negative (provided the bank is solvent before undertaking the investment opportunity).

[^12]:    ${ }^{3}$ The effect of $T$ is only on the tax benefit of debt and not the interest rate. Since it is clear that debt savings are increasing in $T$ from (3.6) and 3.11, I omit comparative statics analysis of $T$.

[^13]:    ${ }^{1} r$ is fixed at $9.787 \%$ without illiquidity risk (for the "neither" and "taxes only" curves) and $11.783 \%$ with illiquidity risk (for the "illiquidity risk only" and "both" curves). These correspond to the initial values obtained in Section 4.1 .

[^14]:    ${ }^{1}$ See (Elliot 2014, 5).

[^15]:    ${ }^{2}$ See (Basel Committee 2013) for the full specifications.

[^16]:    ${ }^{3}$ I omit details of how "high quality liquid assets" and "projected cash claims" are defined, as they are not very relevant to the simple balance sheet in my model. Note, however, that Basel III makes assumptions on what percentage of liabilities will "run off" when calculating the projected cash claims that need to be covered by liquid assets, implying that not all liabilities will run off at once. This is at odds with the result of Morris and Shin (2016) used in my model that everyone runs if and only if $\theta_{1}$ falls below the run point $\theta_{0}^{*}$. One can reconcile this inconsistency by, for example, generalizing the model to allow for the existence of many investments, each funded with its own stock of short-term debt. Then the different stocks of short-term debt may "run off" at different run points.

